Iterative PIC Detection and Channel Estimation for Satellite DS-CDMA Communications

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***Abstract*: This paper presents a Turbo Multiuser Detector for Turbo-Coded DS-CDMA systems, based on the utilization of a PIC and a bank of turbo decoders, in which the PIC performs interference cancellation after each constituent decoder of the turbo decoding scheme. Moreover, the soft output of turbo decoders are used iteratively to improve the updating step of the channel parameter estimation which is formally equivalent to one step of the expection-maximization algorithm.**

By means of computer simulations, we will show that the proposed receiver achieves performance comparable with systems which suppose perfect channel parameters knowledge for medium to high system loads, in AWGN channel. These results are obtained also in satellite channel, for different values of the elevation angle.

1. Introduction

Wide-band code division multiple access (CDMA) [1] has been selected as the fundamental signaling technique for third generation mobile communication. One disadvantage of CDMA systems is their vulnerability to Multiple Access Interference (MAI): hence, serial and parallel interference cancellation (SIC and PIC) techniques are particularly attractive because they process directly the outputs of a bank of single-user matched filters. Since the receiver front-end is the same of the conventional single-user detection, these methods can be used to enhance the performance of a conventional base-station receiver when particularly high system loads are considered.

Main performance limitation of SIC and PIC schemes can be identified in the error propagation caused by feeding back erroneous symbol decisions and the imperfect interference cancellation due to non-ideal knowledge of channel parameters as complex amplitudes and delays of the users' multipath channels.

In early works [2], multiuser detection is applied to uncoded transmission and hard decisions are used to remove the detected users from the received segnal. In order to prevent error propagation, the use of soft interference cancellation and iterative schemes has recently been proposed in different forms [3] [4]; particularly, since practical CDMA communications rely on the utilization of error control coding and interleaving, more and more attention has been addressed to coded systems.

After the successful introduction of Turbo codes [5], many works [6], [7], coupled this iterative decoding technique with the multi-users receivers outperforming conventional schemes, nevertheless introducing an high complexity growing with the number of users. A common feature of these algorithms is that single-user SISO decoders provide at each iteration an estimate of *a posteriori* probabilities for the user code symbols, which are used to form the soft estimates of interference to be subtracted from the received signal. As a result, the contribution of each user is effectively subtracted from the signal only if its symbol decisions are sufficiently reliable.

For the sake of errors reduction of the channel parameters estimation, iterative interference cancellation schemes can be combined with iterative parameter estimation in order to improve the estimates with the iterations, as long as the signal is cleaned-up from interference.

In this paper, we propose an iterative multiuser detector based on the utilization of a Parallel Interference Cancellation (PIC) and a bank of Turbo decoders coupled with a low-complexity iterative soft-PIC algorithm for channel parameters estimation. In order to achieve polynomial complexity in the number of users, we apply expectation maximization (EM) algorithm locally [6], i.e. the true *a-posteriori* distribution of the missing data, given the observation and the current parameter estimate, is replaced by the product distribution induced by the a-posteriori marginal probabilities output by the SISO decoders at each receiver iteration. The proposed approach permits to achieve remarkable performance in AWGN channel, also for overloaded system. Moreoever, the proposed solution can be proved to be effective also in Satellite fading channel.

2. System Model

We consider an up-link DS-CDMA communication system with N synchronous turbo-coded users. Timing, carrier phases and spreading sequences of all the users are assumed to be perfectly known at the receiver, in the base station. Each user encodes blocks of information bits $u_k(i)$ with a Parallel Concatenated Convolutional Code (PCCC) and transmits the resulting codewords composed of M coded bits over a common AWGN channel with BPSK modulation. The equivalent baseband received signal can be written as

$$r(t) = \sum_{k=1}^{N} \sqrt{E_{b_k}} \sum_{i=0}^{M-1} c_k(i) \cdot p(t - iT_b) s_k(t - iT_b) + n(t) \quad (1)$$

where:

- T_b is the bit interval;
- E_{b_k} is the *k*th user received energy;

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- $c_k(i) \in \{+1, -1\}$ is the bit transmitted by *k*th user during the *i*th bit period;
- $p(t)$ is the unit-power rectangular pulse shape with duration T_b;
- $s_k(t)$ is the *k*th user unit-power spreading sequence;
- $n(t)$ is an Additive White Gaussian Noise (AWGN) process with double sided spectrum density $\sigma^2 = No/2$ [W/hz].

In the receiver a bank of matched filter is used for despreading. Without loss of generality, we can assume that the first bit-interval is observed. As a result, the output of the *k*th matched filter is given by

$$\begin{aligned} y_k &= \frac{1}{T_b}\int_0^{T_b} r(t)s_k(t)dt \\ &= \sqrt{E_{b_k}}c_k + \sum_{\substack{j=1\\j\neq k}}^{N} \sqrt{E_{b_j}}c_j\rho_{jk} + n_k \end{aligned} \tag{2}$$

where ρ_{jk} is the normalized crosscorrelation coefficient between users *j* and *k* and n_k is the noise Gaussian sample of user *k* with distribution $N(0,\sigma^2)$. The second term in eq.(2) represent the MAI, that has to be cancelled.

3. Satellite Channel Model

The satellite-mobile channel model assumes flat fading and is based on the one proposed by [8]. For a given propagation environment, the channel parameters are clearly dependent on the actual satellite elevation angle Ψ as it is seen by the generic user. The fading process bandwidth depends mainly on the relative speed between the satellite and the mobile user. In the block diagram shown in Fig. 1, $\xi_L(t)$ is a log-normal real process, with bandwidth B_L, characterized by the following parameters

$$\begin{aligned} &E\left\{10\log_{10}\left[\xi_L(t)\right]\right\} \triangleq \mu_L \\ &E\left\{\left(10\log_{10}\left[\xi_L(t)\right]-\mu_L\right)^2\right\} \triangleq \left[\sigma_L\right]^2 \end{aligned} \tag{3}$$

Hence, the parameters take into account the effects of signal shadowing. The signal $\beta_R(t) \triangleq \beta_c(t)+j\beta_s(t)$ is a complex Gaussian process with independent unit-power *I-Q* components having bandwidth B_R,which represents multipath effects, while the parameter R is the Rice factor. The satellite channel is further characterized by the average carrier-to-mutipath ratio (C/M) defined as the ratio between the LOS and the multipath power and the LOS power loss given by:

$$\begin{aligned} &\left[\frac{C}{M}\right][dB] = 10\log_{10} R \\ &\left[\Delta P\right]_{LOS}[dB] = \mu_L + \frac{\left[\sigma_L\right]^2}{20\log_{10} e}. \end{aligned} \tag{4}$$

In Fig. 1 the two channel components $\xi_L(t)$ and $\beta_R(t)$ are shown. It is clear that while ξ_L is very slowly varying, so that its effect can be compensated by a power control system, the other component $\beta_L(t)$, characterized by fast variations results to be the main cause of performance loss.

4. The IC Iterative Receiver and Channel estimator

The iterative cancellator with channel estimation consists of an Interference Cancellation (IC) based Multi-User Detector (MUD) followed by N single-user turbo decoders and of an estimator block which provides channel information to the MUD. Each constituent block iteratively provides soft informations to the others, as shown in Fig 2.

The signal received by the channel is elaborated in the first block which extracts the training sequences of every user from the informative frame, where they have been inserted by the transmitter. In the first multiuser detection iteration, the a-priori information of coded bits is not available, i.e. $L_{ap}(c_k(i)) = 0$, k=1,2,...,N, i=0,1,...M-1. The IC stage delivers interference-cancelled soft outputs $\tilde{y}_k(i)$ to the input of the turbo decoders. After a fixed number of turbo decoder iterations, the extrinsic information of coded bits at the output of turbo decoders are fed back to the input of the IC detector as the a priori information for the next receiver iteration and to the channel estimator to upgrade the channel parameters values.

The considered turbo codes are composed of two Recursive Systematic Convolutional (RSC) codes linked by an interleaver and a MAP based algorithm is used for iterative decoding [9]. Since the IC receiver requires soft information about reliability of both the systematic and the parity bits, the decoding algorithm is properly modified to produce also extrinsic information about the latter [10]. At each new iteration, the iterative structure permits the channel estimator and the multiuser receiver to have a more reliable a-priori information and the decoders to operate on soft inputs, in which a greater amount of interference has been cancelled.

4.1. The Iterative PIC Receiver

In the conventional iterative Parallel Interference Cancellation (PIC) receiver [11], at each IC stage, MAI is to be removed simultaneously from each user. Therefore, at the mth receiver iteration, the PIC soft output, i.e., the turbo decoders input, can be expressed as

$$\begin{aligned} \tilde{y}_k^{(m)} &= y_k - \sum_{\substack{j=1\\j\neq k}}^{N} \sqrt{E_{b_j}}\rho_{kj}\hat{c}_j^{(m)} = \sqrt{E_{b_k}}c_k + \\ &\sum_{\substack{j=1\\j\neq k}}^{N} \sqrt{E_{b_j}}\rho_{kj}\left(c_j - \hat{c}_j^{(m)}\right) + n_k \end{aligned} \tag{5}$$

where $\hat{c}_j^{(m)}$ is the estimate of bit c_j at iteration m. Note that the second summation represents the residual MAI after cancellation.

The decision $\hat{c}_k^{(m)}$, for kth user at the mth receiver iteration, is taken as the expectation of c_k, given the channel output and the a priori probability, i.e., [12]

$$\begin{aligned} \hat{c}_k^{(m)} &= \mathrm{E}\left\{c_k \middle| y_k, \mathrm{P}(c_k)\right\} \\ &= \sum_{c_k\in\{+1,-1\}} c_k\, \mathrm{P}\left(c_k \middle| \tilde{y}_k, \mathrm{P}(c_k)\right) \end{aligned} \tag{6}$$

Making the assumption that the interference can be con-

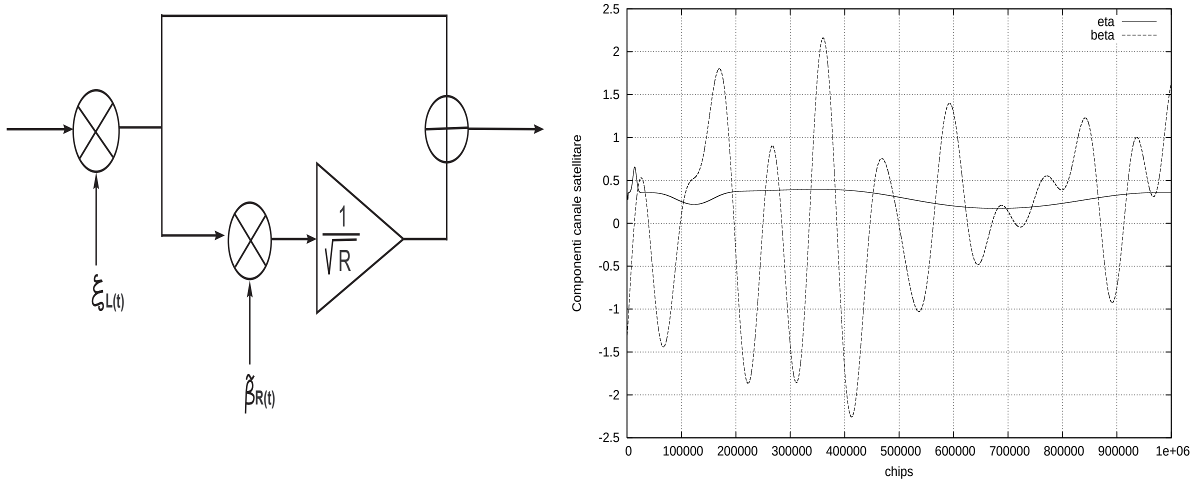

Figure 1: Satellite channel model and components.

sidered as a Gaussian process, we obtain

$$\hat{c}_k^{(m)} = \tanh\left[\frac{1}{2}\left(2\,\frac{\sqrt{E_{b_k}}y_k}{\sigma_k^2} + \mathrm{L}_{ap}^{(m)}(c_k)\right)\right] \qquad (7)$$

where σ_k^2 is the termal noise-plus-interference variance, given by $\sigma_k^2 = \sigma^2 + \sigma_{k,MAI}^2$. The term $\mathrm{L}_{ap}^{(m)}(c_k)$ is the a priori *Log-Likelihood Ratio* of bit c_k at the mth iteration, defined as

$$\mathrm{L}_{ap}^{(m)}(c_k) \triangleq \log\frac{P^{(m)}(c_k = +1)}{P^{(m)}(c_k = -1)}\ . \qquad (8)$$

In the first receiver iteration no a priori information is available from the decoder output: hence, for the initializing condition, it is assumed $\mathrm{L}_{ap}^{(0)}(c_k)$=0, k=1,2,...N. Instead, in the successive iterations the extrinsic information coming from the decoders can be used, leading to $\mathrm{L}_{ap}^{(m)}(c_k)$=$\mathrm{L}_{ex}^{(m-1)}(c_k)$. As it is shown in [13], combination of channel output and extrinsic information in decision statistic yields a biased residual interference term which tends to cancel the useful signal. However, computer simulations confirm that better performance is achieved by using all the information sources and that mitigation of the bias effect is obtained after few iterations.

4.2. Estimation of the User Complex Amplitudes

As seen before, the system is frame-oriented, i.e., encoding and decoding is performed frame-by-frame and users are synchronous also at frame level. The insertion of the training sequence in each frame takes to obtain frames whose lenght, in symbol, is equal to $L+T$, where L and T denote the code block lenght and the training sequence lenght. We assume also that the channel parameters remain constant over each frame; the reason for adopting this simple model is that it is quite realistic in systems like universal mobile telecomunication system (UMTS) division duplex (TDD).

Let $\mathbf{w} = (w_1, \cdots, w_N)^T$ denote the vector of complex ampitudes to be estimated, where N denotes the number of users and T the transpose. The ML estimate of $\mathbf{w}$, given the observed signal $\mathbf{Y}$, is given by:

$$\mathbf{w}^{ML} = \arg\max_{\mathbf{w}} \log p(\mathbf{Y}|\mathbf{w}) \qquad (9)$$

where $p(\mathbf{Y}|\mathbf{w})$ is the conditional pdf of the observed signal given by:

$$p(\mathbf{Y}|\mathbf{w}) \propto \sum_{\mathbf{X}} p(\mathbf{Y}|\mathbf{X},\mathbf{w})\,\Pr(\mathbf{X}|\mathbf{w}) \propto \sum_{\mathbf{x}^1 \in \mathcal{C}_1} \cdots \sum_{\mathbf{x}^N \in \mathcal{C}_N} \exp\left(-\frac{1}{N_0}\sum_{l=1}^{L}|\mathbf{y}_l - \mathbf{S}\chi_l\mathbf{w}|^2\right) \qquad (10)$$

where we have defined:

- $\mathbf{Y} \in \mathbf{C}^{SF\times L}$ is the array of received signal samples;
- $\mathbf{X} \in \mathbf{C}^{N\times L}$ is the array of transmitted code simbol;
- SF denotes the spreading factor;
- $\mathbf{x}^l$ is the code word of the user lth belonging to the code book $\mathcal{C}_l$ of the lth user;
- N_0 is the noise variance;
- $\mathbf{S} \in \mathbf{C}^{SF\times N}$ contains the user spreading sequences by columns;
- $\mathbf{y}_l$ is the received signal vector in the lth symbol interval;
- χ_l is defined as the diagonal matrix composed by the elements $\chi_l = diag(x_{1,l}, \cdots, x_{N,l})$, with $x_{n,l}$ transmitted simbol of user n in the lth symbol interval;

From (10) it is clear that direct ML estimation of $\mathbf{w}$ is infeasible in any practical case, as it has complexity proportional to the total number of user code words $\prod_{n=1}^{N}|\mathcal{C}_n|$.

We assume that the estimate $\hat{\mathbf{w}}^{(m)}$ and the APP $\Pr(\mathbf{X}|\mathbf{Y},\hat{\mathbf{w}}^{(m)})$ are available at mth iteration. Then, we can produce an updated estimate $\hat{\mathbf{w}}^{(m+1)}$ for next iteration by following the EM approach. In the language of EM algorithm [14], $\mathbf{Y}$, $\mathbf{X}$ and $\{\mathbf{Y}, \mathbf{X}\}$ play the role of *incomplete, missing*, and *complete* data. The EM update consist of computing the expected log-likelihood function of the complete data conditionally on the incomplete data and on the current parameter estimate (E-step), and maximizing the result with respect to the parameter (M-step).

Applying the EM step locally, as shown in [6], we find the approximation

$$\hat{\mathbf{w}}^{(m+1)} = \frac{1}{N}\bar{\mathbf{r}} \tag{11}$$

with $\bar{\mathbf{r}}$ defined as:

$$\bar{\mathbf{r}} = \sum_{l=1}^{L} \bar{\chi}_l \mathbf{S}^H \mathbf{y}_l \tag{12}$$

where $\bar{\chi}_l = diag(\overline{x_{1,l}}, \overline{x_{2,l}}, \cdots, \overline{x_{N,l}})$ and $\overline{x_{n,l}}$ denote the first moments of the joint *a-posteriori* pmf $\Pr(\mathbf{X}|\mathbf{Y}, \hat{\mathbf{w}}^{(m)})$ given by:

$$\overline{x_{n,l}} = \sum_{\mathbf{X}} x_{n,l} \Pr\left(\mathbf{X}|\mathbf{Y}, \hat{\mathbf{w}}^{(m)}\right) \tag{13}$$

Notice that (11) is directly computed from the bank of single user matched filter (SUMF) outputs, since $\bar{\mathbf{r}}$ depends on the observed signal $\mathbf{Y}$ only through the SUMF outputs $\mathbf{s}_n^H \mathbf{y}_l$.

4.3. Initialization with the Training Phase

The overall iterative soft-PIC algorithm needs a sufficiently reliable initial estimate $\hat{\mathbf{w}}^{(0)}$ of the complex user amplitudes. For the sake of initialization, a joint ML estimate is obtained from the training phase. This is readly given by

$$\hat{\mathbf{w}}^{(t)} = \left(\mathbf{R}^{(t)}\right)^{-1} \mathbf{r}^{(t)} \tag{14}$$

where $\hat{\mathbf{w}}^{(t)}$ denote the user complex amplitudes vector computed by the training sequences and $\mathbf{r}^{(t)}$ and $\mathbf{R}^{(t)}$ are given respectively by

$$\mathbf{r}^{(t)} = \sum_{t=1}^{T} \chi_t \mathbf{S}^H \mathbf{y}_t = \sum_{t=1}^{T} \begin{bmatrix} x_{1,t}^{(t)} \mathbf{s}_1^H \mathbf{y}_t \\ x_{2,t}^{(t)} \mathbf{s}_2^H \mathbf{y}_t \\ \vdots \\ x_{N,t}^{(t)} \mathbf{s}_N^H \mathbf{y}_t \end{bmatrix} \tag{15}$$

and

$$\mathbf{R}^{(t)} = \sum_{t=1}^{T} \chi_t \mathbf{S}^H \mathbf{S} \chi_t \tag{16}$$

with $x_{l,t}^{(t)}$ as the known training symbols. If the training sequences are mutually orthogonal, i.e. $(\mathbf{X}^{(t)})^H \mathbf{X}^{(t)} = T\mathbf{I}$, we obtain $\mathbf{T}^{(t)} = T\mathbf{I}$ and so no matrix inverse is needed in 14. The receiver is initialized with $\hat{\mathbf{w}}^{(0)} = \hat{\mathbf{w}}^{(t)}$. Then, in next iterations, it exploits the update estimate $\hat{\mathbf{w}}$ provided by the EM step shown in last paragraph.

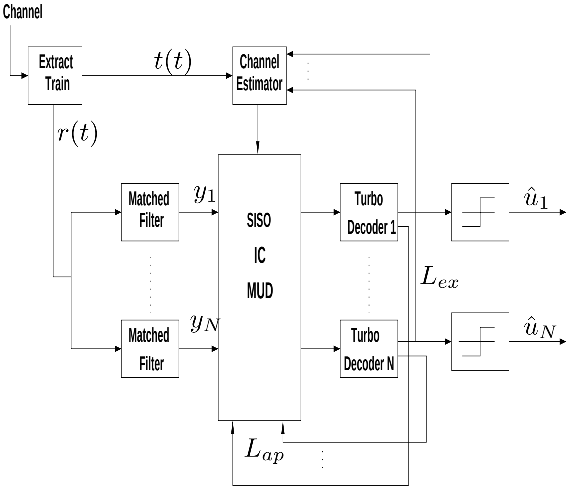

Figure 2: The general iterative IC receiver with channel estimation

5. Simulation Results

In order to demonstrate the performance of the proposed receiver, we considered the following simulation settings, strictly inspired by the UMTS-TDD system:

- Spreading factor $SF = 16$;
- Rate $R_c = 1/2$ turbo code, composed by two 8-state RSC codes with generator polynomials $G_0 = (13)_8 G_1 = (15)_8$;
- Code block lenght $L = 1600$ coded symbols, corresponding to 800 information bits per frame;
- Training sequence lenght $T = 32$;

First, we analyze the performance in a synchronous AWGN channel. The system has 10 equal-power users and the channel complex amplitudes are given by $w_n = \sqrt{R_c E_b} e^{j\phi_n}$, where E_b is the energy per information bit and ϕ is a uniformly distributed random variable over $[-\pi, \pi]$, indipendently generated for each user. Moreover we consider a fixed number of PIC iterations equal to 18 to study the asymptotic behaviour. In Fig. 3 is shown the a comparison of performance vs E_b/N_0 ratio in case of perfect knowledge of channel parameters (Ideal), of channel estimation with one turbo-decoder iteration (TDec), with three TDec iteration and with MAI cancellation from the received training symbols. In the two last cases we almost obtain the same excellent performance, but TC system requires lower computational complexity. Referring to the previous system parameters configuration, Fig. 4, Fig. 5, Fig. 6 and Fig. 6 show the Mean Square Error and the estimator Variance for the amplitude and phase for the 10 users system: from these figures it is evident that the proposed estimation techniques permit remarkable parameter estimation.

Then, we consider the satellite channel model seen in section III in which we used the following values: log-normal shadowing bandwidth $B_L = 0.8Hz$, fading bandwidth $B_R = 50Hz$, elevation angles of 60 and 20 degrees, best and worst cases respectively for 4 active users. In order to follow the channel variations we divide the training sequences on all the frame lenght. The relative results are shown in Fig. 8 and Fig. 9: particularly, the 32 training sequences are divided in 4 and 8 parts: as it is evident, better performance is achieved when the training sequence is divided in 4 parts.

6. Conclusion

In this paper a Turbo-Multiuser Detector and channel estimator has been presented. The proposed receiver uses the soft output of turbo decoders are used iteratively to improve the channel parameters estimation.

By means of computer simulations, it has been shown that the proposed receiver achieves performance comparable with systems which suppose perfect channel parameters knowledge for medium to high system loads, in AWGN channel. Remarkable results are obtained also in satellite channel, for different values of the elevation angle.

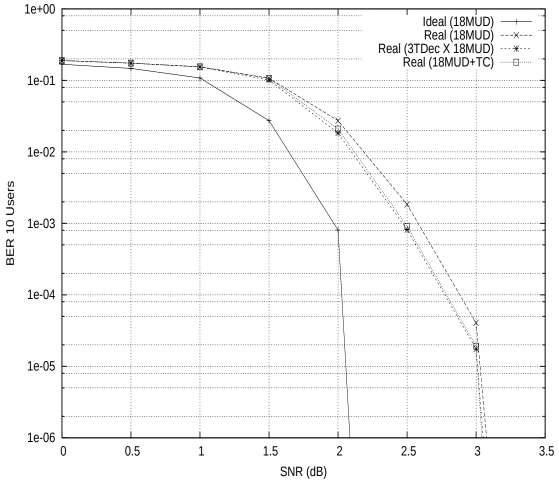

Figure 3: Performance comparison of iterative PIC receiver with Ideal and Real channel parameters knowledge, 10 equal-power users.

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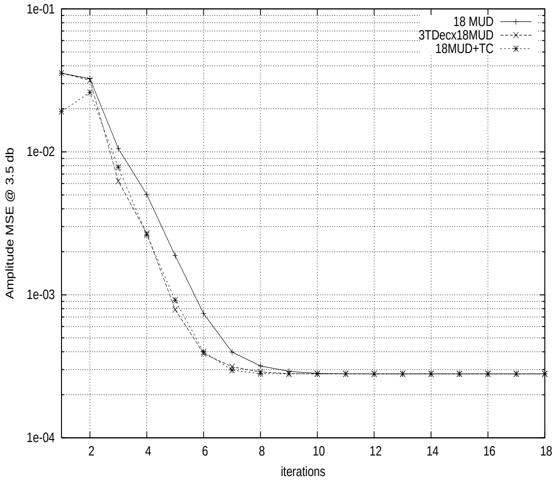

Figure 4: Amplitude MSE in AWGN channel, 10 equal-power users at 3.5 dB.

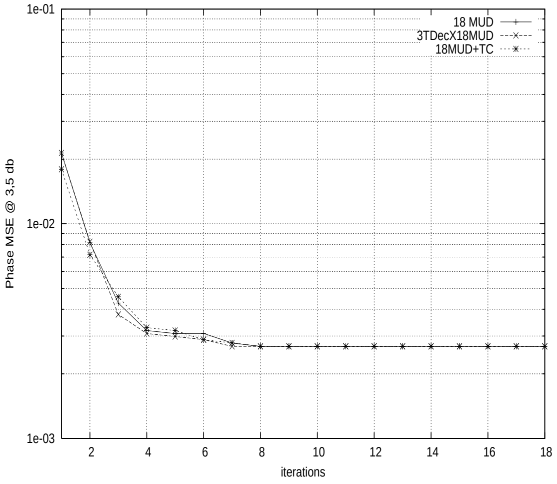

Figure 5: Phase MSE in AWGN channel, 10 equal-power users at 3.5 dB.

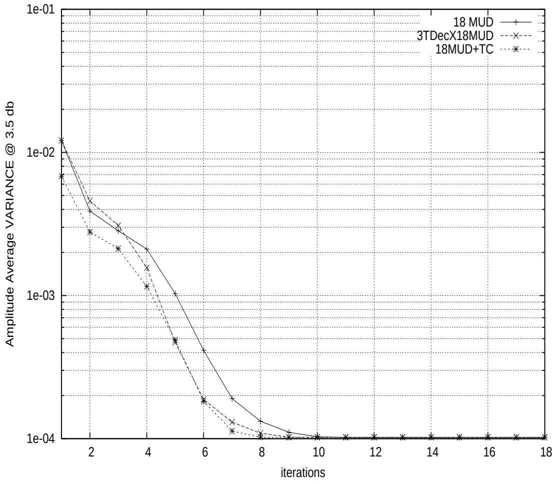

Figure 6: Amplitude System Variance in AWGN channel, 10 equal-power users at 3.5 dB.

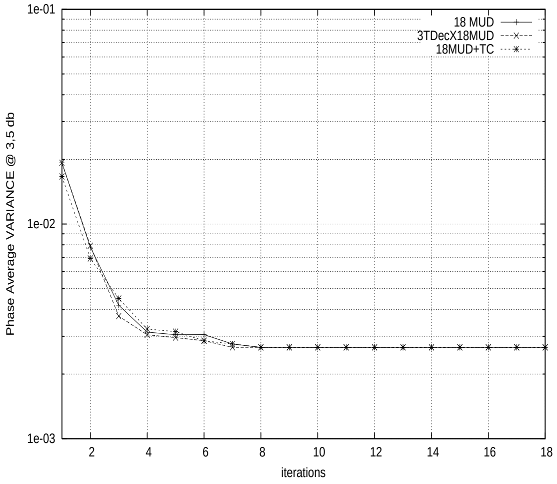

Figure 7: Phase System Variance in AWGN channel, 10 equal-power users at 3.5 dB.

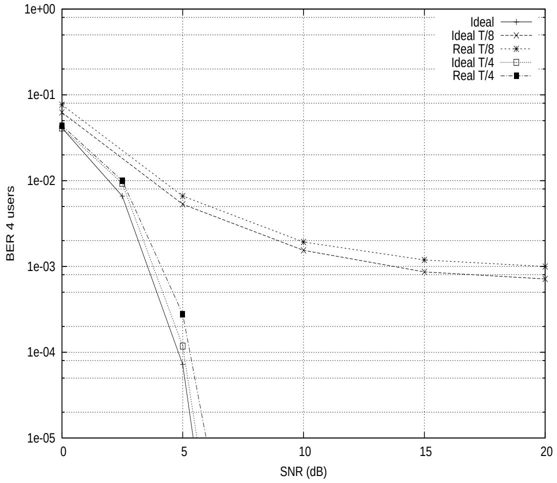

Figure 8: Performance of iterative PIC with Ideal and Real channel parameters in satellite contest, 4 equal-power users, elevation angle equal to $60°$.

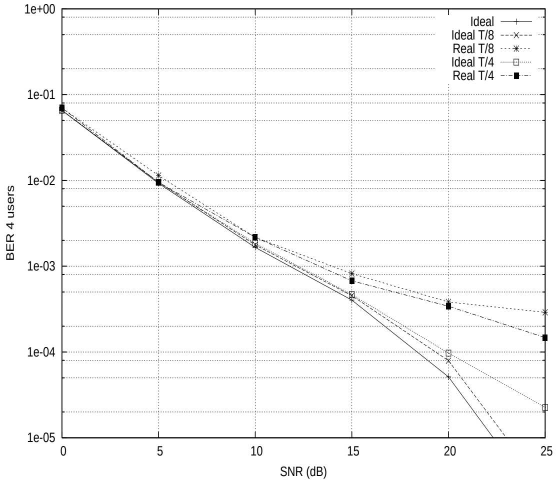

Figure 9: Performance of iterative PIC with Ideal and Real channel parameters in satellite contest, 4 equal-power users, elevation angle equal to $20°$.