Worst-Case Traffic in a Tree Network of ATM Multiplexers

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***Abstract*—In this paper we study tree networks of discrete-time queues loaded with periodic traffic sources. By using the so-called Beneš method, exact closed-form expressions are obtained for the queue length distributions. The models developed can be used to study the superposition of periodic sources emitting bursts of cells in ATM networks. The results obtained show the significant effect that this kind of traffic can have on the performance of these systems.**

***Index Terms*—ATM, Beneš method, periodic traffic, tree networks, worst-case traffic.**

I. INTRODUCTION

IN THIS paper exact closed-form formulas are obtained for the queue length distributions in a discrete-time M-stage tree queueing network loaded with periodic traffic sources. This kind of network is defined in the following way: We have a number of discrete-time queues with service time equal to one time-slot. These queues are grouped into M groups or stages. Each queue of a stage is fed by *all* the output traffic of any given number (which could be 0) of queues from the previous stage as well as by a certain number (which could be 0) of external traffic sources. *All* the input traffic in the network is routed to the root queue, which occupies the first stage [see Fig. 2(a) for an example of case $M = 3$].

Ref. [10] shows that only two configurations are relevant for finding queue length distributions in this kind of network: case $M = 1$, which is for a single server queue with a constant service time, and case $M = 2$, which is for a two-stage tree network. Once the solution for these cases is known, the solution for the general case $M > 2$ can be found solving systems with $M = 1$ and $M = 2$.

The traffic sources are periodic and independent. In one period the input traffic sources emit a burst of back-to-back clients (cells in ATM terminology) of constant length equal to one time slot. Burst lengths can be different for each traffic source, but they all have a common period T.

The results presented in this paper are in fact a generalization of the results obtained in [13] for a case in which the periodic input consists of bursts of back-to-back cells and for a case in which the system consists of a tree network of queues.

The models studied in this paper are of interest, for example, in the context of packet networks offering quality-of-service (QoS) guarantees, as is the case of ATM networks. The congestion control for real-time traffic in ATM networks is based on an open-loop mechanism. The steps taken for establishing a connection are (see [1]):

1) At the connection set-up stage the user and the network negotiate a traffic contract. This contract specifies the characteristics of the connection agreed on. At the public user-network interface the traffic contract consists of the selected ATM service categories (e.g., constant bit rate, variable bit rate), the set of QoS parameters (e.g., cell transfer delay, cell loss ratio), the source traffic descriptor (e.g., peak cell rate, sustainable cell rate), the cell delay variation tolerance, and the conformance definition.
2) The connection admission control (CAC) determines whether the new connection can be accepted in the network, given the load conditions and the QoS requested.

If the new connection is accepted, it is policed by the user parameter control function in order to check whether the connection is compliant with the traffic contract established at connection set-up.

A well-known problem that arises with this congestion control scheme is caused by the tolerances to be introduced in the user parameter control function. The simplest example is given in the case of constant bit rate (CBR) connections: given a traffic contract established for a CBR connection, there are different traffic patterns which conform to the contract but which produce different effects on network congestion. For instance, if the traffic contract declares particular values for peak cell rate (PCR) and cell delay variation tolerance (CDVT), the following two different traffic patterns will comply with the traffic contract declared:

1) A "pure" CBR connection, i.e., a traffic pattern consisting of a single ATM cell emitted periodically each $1/\mathrm{PCR}$ time-slots.
2) A worst-case traffic (WCT) pattern, i.e., a traffic pattern consisting of periodic bursts of b cells emitted each b/PCR slots, where b is given by

$$b = 1 + \left\lceil \frac{\mathrm{CDVT} \times \mathrm{PCR}}{1 - \mathrm{PCR}} \right\rceil \tag{1}$$

As will be seen from the results given in this paper, the presence of WCT patterns can have a huge impact on the network performance.

Note that the so-called WCT is not exactly the *worst*-case traffic that can enter the queue while still complying with the traffic contract. Other traffic patterns can have slightly worse

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effects on the queue length [12]. However, they are constructed in a much more artificial way, and thus we consider that the adverse network conditions which may arise due to tolerances in the policing function can be studied better by using the previously defined WCT.

Another scenario which can lead to degraded network performance will appear through the existence of correlations between different connections. The models developed in this paper allow the effect of these correlations to be studied in certain special cases. More specifically, if certain sources emit periodic traffic with the same period in different virtual channels (VCs) but synchronize the emission of these cells, there will again be periodic traffic with arrivals in bursts.

Several articles have dealt with the case of multiplexing periodic traffic in a single queue. Some of the techniques most commonly used have been the Beneš method [13] and the ballot theorem [6]. In [12] there is a full description of the Beneš method. In [3] and [11] the superposition of CBR sources in a single queue is also studied. Refs. [7], [8] and [2] address the multiplexing of traffic sources that deliver packets periodically according to an arbitrary i.i.d batch arrival process, known as non-deterministic periodic traffic. Note that WCT is a particular case of this class of process. These models are of interest, for example, in the context of the statistical multiplexing of traffic that arrives over access lines that may have lower capacity than the trunk and are solved through the introduction of a discrete-time Markov chain. In [2] a more complete reference list of papers on the topic is given. To our knowledge the case of several queues in discrete-time with periodic traffic has not previously been addressed. Ref. [9] gives an expression for the average queue length in a tree network loaded with Poisson traffic.

When the sources do not have the same period it seems to be much more difficult to obtain an exact solution. For the case of a single queue, [13] gives upper and lower bounds of the queue length probability distribution function. The techniques described in [7], [8] and [2] can be used to solve this case, although with an increasing numerical complexity. The general case of a network of queues in discrete time in which a tree topology is not imposed also seems a much more difficult matter to approach.

The paper is organized as follows: Section II gives exact closed-form expressions for the queue length distribution of a single queue system; Section III examines the two-stage system and Section IV studies the extension to the M-stage system; Section V provides some numerical examples, and last, Section VI gives the conclusions.

II. MULTIPLEXING PERIODIC SOURCES IN SINGLE QUEUE

In this section the so-called Beneš method is used to obtain an exact and closed-form formula for the queue length probability distribution function of a multiplexer with a service time equal to one time slot loaded with N independent and periodic sources. Departures are considered to take precedence over arrivals (i.e., first comes the service (if any), then there are the cell arrivals (if any), and finally the system is observed). To make the paper more complete, the Appendix includes the derivation, using the Beneš method, of the expression of the queue length probability distribution function for the general case of a G/D/1 queueing system presented in [13].

We assume that we have N independent and periodic sources of common period T. Sources can be classified into I groups depending on their burst length b_i. The ith group has N_i sources with burst length b_i and period T. Using vector notation $\boldsymbol{N} \equiv (N_1, \ldots, N_I)$ and $\boldsymbol{b} \equiv (b_1, \ldots, b_I)$. A stable system is assumed (i.e., $(\boldsymbol{N} \cdot \boldsymbol{b}/T) < 1$).

From (25) in the Appendix the following expression for the queue length complementary probability distribution function (CPDF) is obtained:[1]

$$Q(T, \boldsymbol{N}, \boldsymbol{b}, x) \equiv P\{L_0 > x\} = \sum_{t=1}^{\boldsymbol{N}\cdot\boldsymbol{b}-x} \sum_{\substack{0\le \boldsymbol{n}\cdot\boldsymbol{1}\le N\\ 0\le n_i\le N_i}} \cdot P\{\mathcal{A}(t) = t+x,\, L_{-t} = 0,\, \boldsymbol{B}(t) = \boldsymbol{n}\} \tag{2}$$

where the following definitions are used:

1) $\mathcal{A}(t)$ is the number of cell arrivals during interval $(-t, 0]$ and L_{-t} is the queue length at time slot $-t$.
2) $\boldsymbol{B}(t) \equiv (B_1(t), \ldots, B_I(t))$; $B_i(t)$ being the number of burst arrivals produced by group i sources during the interval $(-t, 0]$.
3) $\boldsymbol{n} \equiv (n_1, \ldots, n_I)$ and $\boldsymbol{1} \equiv (1, \ldots, 1)$.

Note that $\mathcal{A}(t)$ includes the cells originated by bursts that started during $(-(t+b_i), -t]$. However if a burst is active during time slot $-t$ the queue cannot be empty at time slot $-t$. Let us define $A(t)$ as the number of cell arrivals during an interval $(-t, 0]$ *that are originated by bursts that start during the same interval*. It can be easily seen that the two events $\{\mathcal{A}(t) = t+x,\, L_{-t} = 0,\, \boldsymbol{B}(t) = \boldsymbol{n}\}$ and $\{A(t) = t+x,\, L_{-t} = 0,\, \boldsymbol{B}(t) = \boldsymbol{n}\}$ are identical, and thus

$$\begin{aligned} Q(T, \boldsymbol{N}, \boldsymbol{b}, x) &\\ \equiv P\{L_0 > x\} &\\ = \sum_{t=1}^{\boldsymbol{N}\cdot\boldsymbol{b}-x} \sum_{\substack{0\le \boldsymbol{n}\cdot\boldsymbol{1}\le N\\ 0\le n_i\le N_i}} & A(t, t+x, \boldsymbol{n}) L(t, \boldsymbol{n}) B(t, \boldsymbol{n}) \\ = \sum_{t=1}^{\boldsymbol{N}\cdot\boldsymbol{b}-x} \sum_{n_1=0}^{N_1} \cdots \sum_{n_I=0}^{N_I} & A(t, t+x, \boldsymbol{n}) L(t, \boldsymbol{n}) B(t, \boldsymbol{n}) \end{aligned} \tag{3}$$

where

1) $A(t, m, \boldsymbol{n}) \equiv P\{A(t) = m | L_{-t} = 0, \boldsymbol{B}(t) = \boldsymbol{n}\}$;
2) $L(t, \boldsymbol{n}) \equiv P\{L_{-t} = 0 | \boldsymbol{B}(t) = \boldsymbol{n}\}$;
3) $B(t, \boldsymbol{n}) \equiv P\{\boldsymbol{B}(t) = \boldsymbol{n}\}$.

As explained in the Appendix, the previous formula is for an auxiliary queue system in which bursts that start for $t \le -T$ are switched off [this is the reason why $A(t)$ is used instead of $\mathcal{A}(t)$]. Note that the queue length distributions for $t = 0$ in the original system and in the auxiliary system are identical. However, this is *not* the case for the queue length distributions for $t < 0$.

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[1] $P\{\mathcal{A}(t) = t+x\} = 0$ if $t > \boldsymbol{N} \cdot \boldsymbol{b} - x$.

In the remainder of this section exact expressions are obtained for each of the terms appearing in the above formula.

When the period T and the number of sources N becomes larger the calculation complexity for evaluating the above formula also increases.[2] In this case the behavior of the system can be approximated by that of a multiplexer loaded with a source which emits bursts of lengths b_i. The number of class ith bursts which start during a particular time interval is given by Poisson distribution of parameter λ_i ($\mathbf{\Lambda} \equiv (\lambda_1, \ldots, \lambda_I)$, $(\boldsymbol{N}/T) \to \mathbf{\Lambda}$). This system is referred to as a "Poisson approximation." The expression for the CDPF function of the queue length is now

$$\begin{aligned} Q(\mathbf{\Lambda}, \boldsymbol{b}, x) \equiv & P\{L_0 > x\} \\ = & \sum_{t=1}^{\infty} \sum_{n_1=0}^{\infty} \cdots \sum_{n_I=0}^{\infty} A(t, t+x, \boldsymbol{n}) \\ & \cdot L(t, \boldsymbol{n}) B(t, \boldsymbol{n}). \end{aligned} \tag{4}$$

A. Term $A(t, t+x, \boldsymbol{n})$

With independent sources we can express $A(t, t+x, \boldsymbol{n})$ as

$$A(t, t+x, \boldsymbol{n}) = \sum_{m_1+\cdots+m_I=t+x} \prod_{i=1}^{I} A_i(t, m_i, n_i) \tag{5}$$

where $A_i(t, m, n)$ is defined as

$$A_i(t, m, n) = P\{A_i(t) = m | L_{-t} = 0, B_i(t) = n\}. \tag{6}$$

Let us define $a_i(j, t)$ as the probability of a *single* class i source which begins its active period during interval $(-t, 0]$ emitting j cells during this interval ($0 < t \le T$ and $0 < j \le b_i$). The expression of $a_i(j, t)$ can readily be obtained as

1) If $j < b_i$ and $t \ge j$ then $a_i(j, t) = (1/t)$.
2) If $j = b_i$ and $t \ge b_i$ then $a_i(b_i, t) = (t - b_i + 1)/t$.
3) $a_i(j, t) = 0$ otherwise.

To obtain an expression for the term $A_i(t, m, n)$ two regions depending on the value of t are considered, as follows:

1) *Region I:* If $t < b_i$, all the bursts will partly contribute to the next period. The probability distribution for the number of cells produced by n class i sources contributing in $(-t, 0]$ is given by the n fold convolution of function $a_i(j, t)$, $a_i^{(n)}(j, t)$.
2) *Region II:* If $b_i \le t < T$, bursts that become active in $(-T, -(b_i - 1)]$ will contribute with all their cells, whereas bursts that become active in $(-(b_i - 1), 0]$ will contribute only partially.

   Note that if $T - b_i \le t < T$, the arrival of a burst during $(-T, -t]$ would not produce a zero queue length at time $-t$. Every source must therefore initiate its burst during $(-t, 0]$.

From the above we obtain

$$A_i(t, m, n) = \begin{cases} \dfrac{1}{t^n} u_t^{(n)}(m) & \text{if } 0 < t < b_i,\ n \le N_i \\ \displaystyle\sum_{j=0}^{n} \binom{n}{j} \frac{(t - b_i + 1)^j}{t^n} \\ \quad \cdot u_{b_i-1}^{(n-j)}(m - jb_i) & \text{if } b_i \le t \le T - b_i,\ n \le N_i \\ \displaystyle\sum_{j=0}^{N_i} \binom{N_i}{j} \frac{(t - b_i + 1)^j}{t^{N_i}} \\ \quad \cdot u_{b_i-1}^{(N_i-j)}(m - jb_i) & \text{if } T - b_i \le t < T,\ n = N_i \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where function $u_c(y) = 1$ in the interval $[1, c]$ and $u_c(y) = 0$ otherwise. $u_c^{(n)}(y)$ denotes the n fold discrete-time convolution of $u_c(y)$. A simple expression for $u_c^{(n)}(y)$ is (see [4], [5])

$$u_c^{(n)}(y) = \sum_{s=0}^{\lfloor (y-n/c) \rfloor} (-1)^s \binom{n}{s} \binom{y - s \cdot c - 1}{n - 1} \quad \text{if } n > 0 \text{ and } y = n, \ldots, nc \tag{8}$$

$u_c^{(n)}(y) = 0$ for other values of y. For $n = 0$ we define $u_c^{(0)}(y) = \delta(y - c)$, where $\delta(y)$ is the unitary discrete-time Dirac function.

In the case of Poisson approximation, the above expression continues to be valid, although in this case only the first two conditions can be fulfilled.

B. Term $L(t, \boldsymbol{n})$

To obtain an expression for $L(t, \boldsymbol{n})$ arguments similar to those used to solve the $N \cdot D/D/1$ queueing system [12], [13] are applied: event $\{L_{-t} = 0 | \boldsymbol{B}(t) = \boldsymbol{n}\}$ is for a situation in which an auxiliary queue loaded with *periodic* arrivals of period T–t is empty at time $-t$. The arrivals at the aforementioned auxiliary queue consist of $N_i - n_i$ independent bursts of b_i cells for $i = 1, \ldots, I$ evenly distributed over the period $(-(T - t), 0]$.

When $(\boldsymbol{N} - \boldsymbol{n}) \cdot \boldsymbol{b} \ge T - t$ this auxiliary queue is unstable, and thus the probability of having an empty queue is 0. On the other hand, if the auxiliary queue is stable, and since $-t$ is an arbitrary instant with respect to the periodic arrival process of the auxiliary queue, $L(t, \boldsymbol{n})$ can be found using the familiar "$1 - \rho$" expression for the probability of an empty queue, arriving to

$$L(t, \boldsymbol{n}) = \begin{cases} 1 - \dfrac{(\boldsymbol{N} - \boldsymbol{n}) \cdot \boldsymbol{b}}{T - t} & \text{if } (\boldsymbol{N} - \boldsymbol{n}) \cdot \boldsymbol{b} < T - t \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Note that term $A(t, t+x, \boldsymbol{n})$ is nonzero only if $\boldsymbol{n} \cdot \boldsymbol{b} \ge t + x$ and that this condition implies $(\boldsymbol{N} - \boldsymbol{n}) \cdot \boldsymbol{b} < T - t$. Hence, product $A(t, t+x, \boldsymbol{n}) L(t, \boldsymbol{n})$ vanishes if $\boldsymbol{n} \cdot \boldsymbol{b} < t + x$.

In the case of the Poisson approximation

$$L(t, \boldsymbol{n}) = 1 - \mathbf{\Lambda} \cdot \boldsymbol{b}. \tag{10}$$

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[2] For example, using an O2 Silicon Graphics computer with OS IRIX Release G.3 IP32 the computation time for $T = 1000$, $b = 8$ and $N = 50$ the CPU time is well below 30 s. If we take $N = 100$, CPU time is around 500 s.

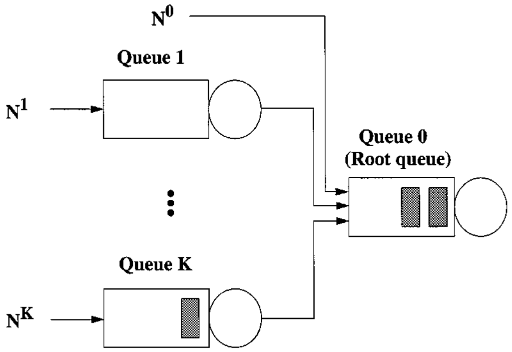

Fig. 1. Two-stage tree network.

C. Term $B(t, \boldsymbol{n})$

Since the sources are independent, term $B(t, \boldsymbol{n})$ can be calculated as

$$B(t, \boldsymbol{n}) = \prod_{i=1}^{I} \binom{N_i}{n_i} \left(\frac{t}{T}\right)^{n_i} \cdot \left(1 - \frac{t}{T}\right)^{N_i - n_i}. \tag{11}$$

The Poisson approximation would lead to

$$B(t, \boldsymbol{n}) = \prod_{i=1}^{I} e^{-\lambda_i t} \frac{(\lambda_i t)^{n_i}}{n_i!}. \tag{12}$$

III. TWO-STAGE TREE NETWORK

In this section the multiplexing of periodic sources in two-stage tree networks (i.e. $M = 2$) is studied. (See Fig. 1.) As in the subsection above, each source is periodic and emits a constant number of back-to-back cells, remaining silent for the rest of the period. We consider pooling K buffers into a single queue, the root queue, numbered as queue 0. Each queue has a constant service time of one time slot.

The kth queue ($k = 0 \ldots K$) receives traffic from N^k sources.[3] Each of these N^k belongs to one of the I source classes. We define $\boldsymbol{N^k} \equiv (N_1^k, \cdots, N_I^K)$, where N_i^k denotes the number of class i sources that directly feed the kth queue.

Stability is assumed at each queue of the tree, i.e., $\boldsymbol{N^k} \cdot \boldsymbol{b} < T$ for $k = 0, \ldots, K$ and $\sum_{k=0}^{K} \boldsymbol{N^k} \cdot \boldsymbol{b} < T$.

We can apply (25) for the root queue, taking into account that $\mathcal{A}^0(t)$ includes cells that arrive directly to the root queue and cells that come from the second-stage queues. For the condition $L_{-t}^0 = 0$ to be true, bursts that directly feed the root queue cannot be active at time $-t$. Also the second-stage queues have to be empty at time $-(t+1)$ (i.e. $L_{-(t+1)}^k = 0, k > 0$).[4] This last condition also implies that bursts that feed the second-stage queues cannot be active at time $-(t+1)$. Let $A^0(t)$ be the number of cell arrivals to the root queue during interval $(-t, 0]$ that are originated by either: 1) bursts that directly feed root queue and become active during $(-t, 0]$; or 2) bursts that feed second-stage queues and become active during $(-(t+1), -1]$. We will have

$$\begin{aligned} \{L_{-t}^0 = 0, \mathcal{A}^0(t) = t + x\} \\ &= \{L_{-t}^0 = 0, L_{-(t+1)}^1 = 0, \cdots, L_{-(t+1)}^K = 0, \\ &\quad A^0(t) = t + x\}. \end{aligned}$$

Defining

1) $\boldsymbol{B^0}(t) \equiv (B_1^0(t), \ldots, B_I^0(t))$ where $B_i^0(t)$ is the number of burst arrivals produced by the class i sources that directly feed the root queue during an interval $(-t, 0]$;
2) $\boldsymbol{B^k}(t) \equiv (B_1^k(t), \ldots, B_I^k(t))$ where $B_i^k(t)$ is the number of burst arrivals produced by the class i sources that directly feed queue k ($k = 1, \ldots, K$) during an interval $(-(t+1), -1]$;
3) $A(t, m, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K}) = P\{A^0(t) = m | L_{-t}^0 = 0, \boldsymbol{B^0}(t) = \boldsymbol{n^0}, L_{-(t+1)}^k = 0, \boldsymbol{B^k}(t) = \boldsymbol{n^k}, k = 1, \cdots, K\}$;
4) $L(t, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K}) = P\{L_{-t}^0 = 0, L_{-(t+1)}^k = 0, k = 1, \ldots, K | \boldsymbol{B^k}(t) = \boldsymbol{n^k}, k = 0, \ldots, K\}$;
5) $B(t, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K}) = P\{\boldsymbol{B^0}(t) = \boldsymbol{n^0}, \ldots, \boldsymbol{B^K}(t) = \boldsymbol{n^K}\}$,

the expression for the queue length CPDF at the root queue will be

$$\begin{aligned} P\{L_0^0 > x\} = \sum_{t=1}^{\sum_{k=0}^{K} \boldsymbol{N^k} \cdot \boldsymbol{b} - x} \sum_{\substack{0 \le \boldsymbol{n^0} \cdot \boldsymbol{1} \le N^0 \\ 0 \le n_i^0 \le N_i^0}} \cdots \sum_{\substack{0 \le \boldsymbol{n^k} \cdot \boldsymbol{1} \le N^K \\ 0 \le n_i^K \le N_i^K}} \\ \cdot A(t, t + x, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K}) L(t, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K}) \\ \cdot B(t, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K}). \end{aligned} \tag{13}$$

In the rest of the section the expressions for each of these terms will be determined. The calculation complexity of the above formula is considerable. This can be reduced if we restrict ourselves to the case in which the sources which directly feed one same queue of the network have the same burst length, though the burst lengths for sources which feed different queues may be different. In this case we do not need to use a vector notation, since for the kth queue the terms of vector $\boldsymbol{N}^k = (N_1^k, \ldots, N_I^k)$ will all be zero except, possibly, for one single value of i. The above formula would therefore be simplified as follows:

$$\begin{aligned} P\{L_0^0 > x\} = \sum_{t=1}^{\sum_{k=0}^{K} \boldsymbol{N^k} \cdot \boldsymbol{b} - x} \sum_{n^0=0}^{N^0} \cdots \sum_{n^K=0}^{N^K} \\ \cdot A(t, t + x, n^0, \ldots, n^K) L(t, n^0, \ldots, n^K) \\ \cdot B(t, n^0, \ldots, n^K) \end{aligned} \tag{14}$$

In any event, like the case for a single queue, the calculation complexity involved with large T periods can be reduced by using a Poisson approximation, defined similarly to the one in section above.

A. Term $A(t, t + x, \boldsymbol{n^o}, \ldots, \boldsymbol{n^K})$

In order to obtain an expression for this term the following notation is introduced:

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[3] We use superscript for the queue number and subscript for the source class.

[4] A nonempty second stage queue at time $-(t+1)$ would imply a cell departure from this queue at time $-t$ and then necessarily $L_{-t}^0 > 0$.

1) $A^{k0}(t)$ is the number of cells that arrive at the root queue from the kth queue during the interval $(-t, 0]$ $(k = 1, \cdots, K)$ and were originated by bursts that became active during $(-(t+1), -1]$.
2) $A_i^{00}(t)$ is the number of cells that arrive at the root queue from the sources that directly feed this queue during the interval $(-t, 0]$ and were originated by bursts that became active during the same time period.
3) $A^{k0}(t, m, \boldsymbol{n^k}) \equiv P\{A^{k0}(t) = m | L^k_{-(t+1)} = 0, \boldsymbol{B^k}(t) = \boldsymbol{n^k}\}$.
4) $A^{00}(t, m, \boldsymbol{n^0}) \equiv P\{A^{00}(t) = m | L^0_{-t} = 0, \boldsymbol{B^0}(t) = \boldsymbol{n^0}\}$.

Term $A(t, t+x, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K})$ will be obtained by means of the convolution of the terms $A^{k0}(t, m, \boldsymbol{n^k})$ $(k = 0, \ldots, K)$:

$$A(t, t+x, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K}) = \sum_{m^0+\cdots+m^K=t+x} \prod_{k=0}^{K} A^{k0}(t, m^k, \boldsymbol{n^k}) \tag{15}$$

Terms $A^{00}(t, m, \boldsymbol{n^0})$ *and* $A^{k0}(t, m, \boldsymbol{n^k})$*:* Term $A^{00}(t, m, \boldsymbol{n^0})$ is defined in the same way as term $A(t, m, \boldsymbol{n})$ in the section above. The expression for the term $A^{00}(t, m, \boldsymbol{n^0})$ is thus given by (5), substituting $\boldsymbol{N^0}$ for $\boldsymbol{N}$.

To calculate term $A^{k0}(t, m, \boldsymbol{n^k})$ we take into account the fact that, since the conditioning event forces the kth second-stage queue to be empty at time $-(t+1)$, the number of cells that arrive to the root queue from kth queue is the same as the number of cell departures during the time interval $(-(t+1), -1]$ in a queue which is empty at time $-(t+1)$ and at which n_i^k independent cell batches of size b_i, uniformly distributed, arrive during this time interval. Note that in this equivalent queueing system we have *batch* arrivals and not *burst* arrivals. However, in both cases the output process of the queue is the same. Note also that we have a transient system which can be potentially unstable. However, as explained in the Appendix, this system can be studied by using the Beneš formulation with only slight variations.

If in this equivalent system, $\boldsymbol{n^k} \cdot \boldsymbol{b}$ cells arrive and m cells are emitted, at time instant $t = -1$ there will be either a queue of length $\boldsymbol{n^k} \cdot \boldsymbol{b} - m + 1$ (in case $\boldsymbol{n^k} \cdot \boldsymbol{b} > m$), or a queue of length 0 or 1 (in case $\boldsymbol{n^k} \cdot \boldsymbol{b} = m$). Hence

$$A^{k0}(t, m, \boldsymbol{n^k}) = \begin{cases} 1 & \text{if } m = 0 \text{ and } \boldsymbol{n^k} = \boldsymbol{0} \\ 0 & \text{if } m = 0 \text{ and } \boldsymbol{n^k} \neq \boldsymbol{0} \\ 0 & \text{if } m > 0 \text{ and } \boldsymbol{n^k} \cdot \boldsymbol{b} < m \\ 1 - Q_0^{batch}(t, \boldsymbol{n^k}, \boldsymbol{b}, 1) & \\ \quad \text{if } m > 0 \text{ and } \boldsymbol{n^k} \cdot \boldsymbol{b} = m & \\ Q_0^{batch}(t, \boldsymbol{n^k}, \boldsymbol{b}, \boldsymbol{n^k} \cdot \boldsymbol{b} - m) & \\ \quad -Q_0^{batch}(t, \boldsymbol{n^k}, \boldsymbol{b}, \boldsymbol{n^k} \cdot \boldsymbol{b} - m + 1) & \\ \quad \text{if } m > 0 \text{ and } \boldsymbol{n^k} \cdot \boldsymbol{b} > m & \end{cases} \tag{16}$$

where $Q_0^{batch}(T, \boldsymbol{N}, \boldsymbol{b}, x)$ is the queue length CPDF at time 0 of a single queue that starts with a zero queue length at time $-T$ and at which $\boldsymbol{N}$ batches of length $\boldsymbol{b}$ arrive during the interval $(-T, 0]$. The expression for $Q_0^{batch}(T, \boldsymbol{N}, \boldsymbol{b}, x)$ is

$$P\{L_0 > x\} = \sum_{t=1}^{T-1} \sum_{n_0=0}^{N_0} \cdots \sum_{n_I=0}^{N_I} \left(1 - \frac{(\boldsymbol{N} - \boldsymbol{n}) \cdot \boldsymbol{b}}{T - t}\right)^+ \cdot \delta(t + x - \boldsymbol{n} \cdot \boldsymbol{b}) \frac{t^{\boldsymbol{n} \cdot \boldsymbol{1}} (T - t)^{(\boldsymbol{N} - \boldsymbol{n}) \cdot \boldsymbol{1}}}{T^N} \prod_{i=0}^{I} \binom{N_i}{n_i}. \tag{17}$$

For the case of Poisson approximation the expressions for $A^{00}(t, m, \boldsymbol{n^0})$ and $A^{k0}(t, m, \boldsymbol{n^k})$ would not change.

B. Term $L(t, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K})$

The event $L^0_{-t} = 0$ implies the events $L^k_{-(t+1)} = 0$, $k = 1, \ldots, K$. Hence

$$L(t, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K}) = P\{L^0_{-t} = 0 | \boldsymbol{B^k}(t) = \boldsymbol{n^k}, k = 0, \ldots, K\} \tag{18}$$

To obtain an expression for this term, a property proved in [10] is used. According to this property, the busy and idle periods of the root queue are the same as those of an equivalent queue loaded directly with the traffic sources of the network (see next section). Note that thanks to this key property, we do not need to know the exact output process of the second-stage queues to solve the system. Consequently, we consider an equivalent single-queue and use the same reasoning as was applied in the above section, to obtain

$$L(t, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K}) = \begin{cases} 1 - \dfrac{\sum_{k=0}^{K} (\boldsymbol{N^k} - \boldsymbol{n^k}) \cdot \boldsymbol{b}}{T - t} & \text{if } \sum_{k=0}^{K} (\boldsymbol{N^k} - \boldsymbol{n^k}) \cdot \boldsymbol{b^k} < T - t \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

Note that term $A(t, t+x, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K})$ is nonzero only if $\sum_{k=0}^{K} \boldsymbol{n^k} \cdot \boldsymbol{b^k} \leq t + x$ and that this condition implies $\sum_{k=0}^{K} (\boldsymbol{N^k} - \boldsymbol{n^k}) \cdot \boldsymbol{b^k} < T - t$.

Hence, product $A(t, t+x, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K}) L(t, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K})$ vanishes if $\sum_{k=0}^{K} \boldsymbol{n^k} \cdot \boldsymbol{b^k} > t + x$.

The Poisson approximation will be

$$L(t, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K}) = 1 - \sum_{k=0}^{K} \boldsymbol{\Lambda^k} \cdot \boldsymbol{b}. \tag{20}$$

C. Term $B(t, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K})$

Since the sources are independent, term $B(t, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K})$ can be calculated as

$$B(t, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K}) = \prod_{k=0}^{K} \prod_{i=1}^{I} \binom{N_i^k}{n_i^k} \left(\frac{t}{T}\right)^{n_i^k} \cdot \left(1 - \frac{t}{T}\right)^{N_i^k - n_i^k}. \tag{21}$$

For the case of Poisson approximation we would have

$$B(t, \boldsymbol{n^0}, \ldots, \boldsymbol{n^K}) = \prod_{k=0}^{K} \prod_{i=1}^{I} e^{-\lambda_i^k t} \frac{(\lambda_i^k t)^{n_i^k}}{n_i^k!}. \tag{22}$$

IV. M-STAGE TREE NETWORK

Once the two-stage system is solved, the results can be generalized for an M-stage tree network. We make use of a property of rooted tree networks with discrete-time single-server queues with unit service time proved by Morrison in [10]. Morrison showed that such networks may be replaced by a single queue, with prescribed input, which has the same output as the queue at the root of the tree. The input traffic of this equivalent system consists of the traffic sources of the network delayed by a constant number of time slots equal to the number of stages they have to cross in order to arrive at the root queue.

In a tree network with M stages, the second-stage queues are also root queues of tree sub-networks (with $M-1$ stages). We can apply the property proved by Morrison and replace the sub-networks which have second-stage queues as roots with their equivalent single-queue systems. Hence, in order to analyze the queue length distribution at the root queue, we can build an equivalent network with two stages. Note that in the case of periodic and independent sources the constant delay introduced in the sources of the equivalent two-stage network does not affect the final result.

For instance, let us consider the three-stage network shown in Fig. 2(a). Queue number 1 is the root queue of a two-stage tree network which consists of queues 1, 3, and 4. We can replace this two-stage sub-network with a single queue which has the same output process. This single queue is fed by $\boldsymbol{N^1}+\boldsymbol{N^3}+\boldsymbol{N^4}$ sources,[5] [see Fig. 2(b)]. The $\boldsymbol{N^3}$ and $\boldsymbol{N^4}$ sources that feed queues 3 and 4 are, in the single queue system, delayed by 1 time slot. This equivalent two-stage network can be used to obtain the queue length distribution at the root queue of the three-stage network.

In conclusion, the analysis of a discrete-time tree network with M stages can be reduced to the analysis of equivalent systems with $M=1$ and $M=2$.

V. NUMERICAL RESULTS

In this section the models developed above are used to assess the impact of periodic sources emitting bursts of cells in the context of ATM network congestion control. Three different scenarios are studied: a single queue system, a two-stage multiqueue system and a tandem queueing system (see Fig. 3).

A. Single Queue System

The first of these scenarios to be studied consists of a single queue system loaded with CBR and WCT sources with a common period T.[6] We are mainly concerned with the impact that WCT traffic has on queue length distributions.

Fig. 4 shows the queue length distribution for a single queue loaded with $N=8$ sources of the same class for different values of burst length b. In all the cases, $T=b\times 10$ time-slots, meaning that the load is constant and equal to 0.8. The curves confirm an important fact observed in [11]: *The quantiles for the queue length distribution are almost linear with* b. This can be seen more clearly in Fig. 5, which shows the 10^{-6}, 10^{-8}, and 10^{-10} quantiles of queue length CPDF for the previous experiment. The maximum queue length values are also given.

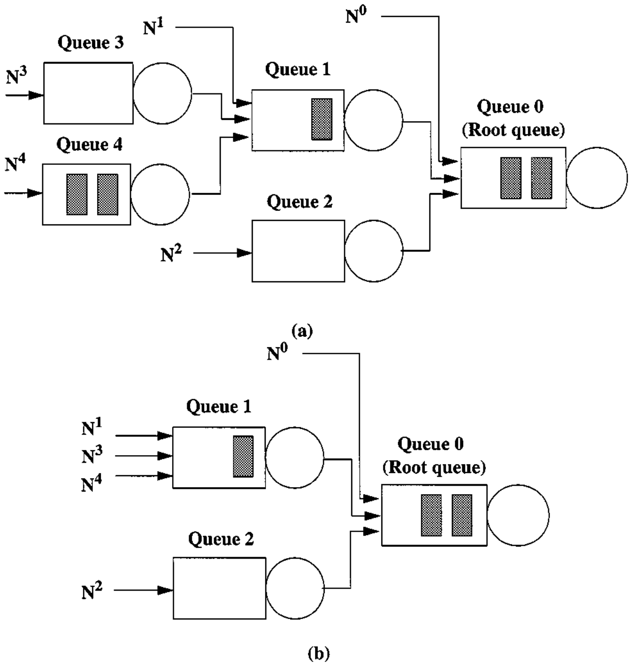

Fig. 2. (a) Example of a three-stage tree network. (b) Equivalent network. Sources N^3 and N^4 are delayed by 1 time-slot.

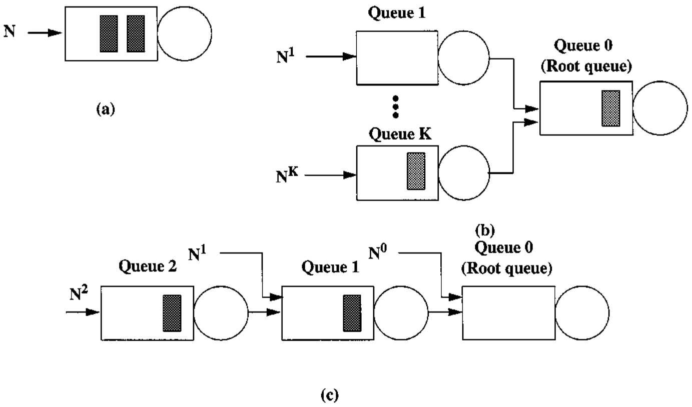

Fig. 3. (a) Single queue system. (b) Two-stage multiqueue system. (c) Three-queue tandem system.

Fig. 6 shows a related effect of the existence of WCT on the network performance, indicating the maximum load values enabling a 10^{-6} quantile of the queue length CPDF of 14 cells to be obtained. The multiplexer is loaded with WCT sources for different values of b. Two curves were obtained, one for the case in which $T=b\times 50$ and another for $T=b\times 100$. We can clearly see how the admissible load quickly drops as the value of b rises.

It is interesting to compare these results with the ones shown in [7], [8] and [2]. In the case of WCT, when the period T is increased, the queue length tail also increases (Fig. 4). For the more general case of non-deterministic periodic traffic sources, two different cases can be distinguished. When the offered traffic in a period T is always smaller than T the same

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[5] Vector notation is used here. For example $\boldsymbol{N^1}$ means that queue 1 is fed by N_1^1 sources of class 1, etc.

[6] Recall that a CBR source was defined as one which periodically emits a single cell, whereas a WCT is one which periodically emits b back-to-back cells in a burst.

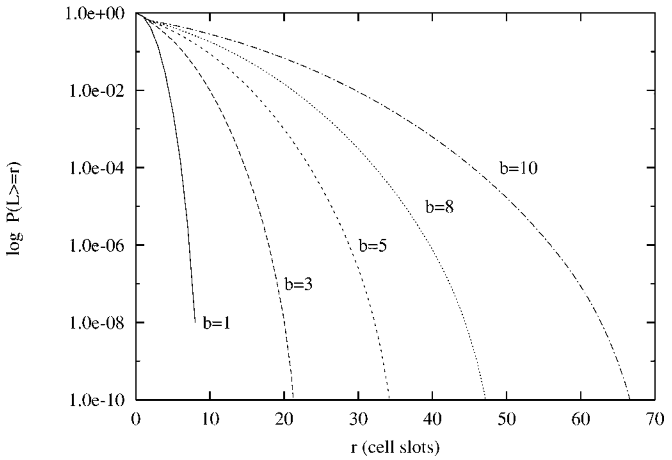

Fig. 4. CPDF for a single queue: homogeneous traffic case, $\rho = 0.8$, $T = b \times 10$, $N = 8$.

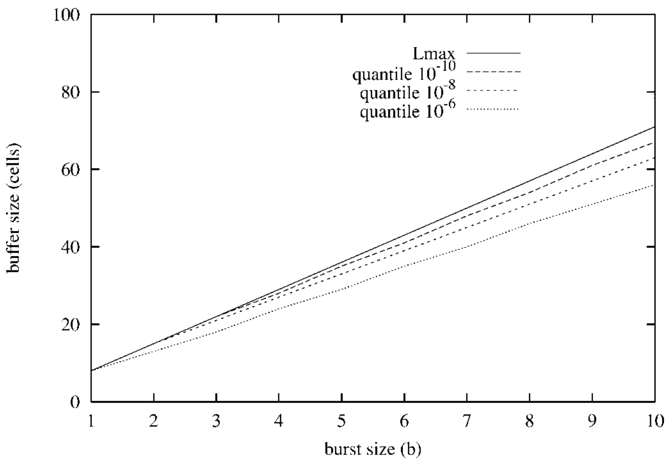

Fig. 5. 10^{-6}, 10^{-8} and 10^{-10} quantiles of the CPDF. The maximum queue length is also shown. Same traffic conditions as in Fig. 4.

behavior is observed (WCT is a particular example of this class of process). However, when the offered load during a period can be greater than T, the queue length tail can be reduced as the period T increases.

In the above examples, homogeneous traffic conditions are assumed, i.e., there are N identical traffic sources. However, it is useful to study the situation in which not every source in the network behaves like a WCT source. Fig. 7 shows the influence of the percentage of WCT sources on the network performance. We show the queue length CPDF when we multiplex N_1 WCT sources, with a burst length of $b_1 = 10$ cells, with N_2 CBR sources with $b_2 = 1$. Every source is considered to have the same period $T = 100$ time slots, and the queue load is kept equal to 0.8. *It is once more seen that the quantiles of queue length CPDF increase linearly with the proportion of WCT.*

This linear increase in the queue length quantiles as a function of b_1 and the percentage of WCT in the input traffic is further illustrated in Fig. 8. This figure shows the 10^{-6} quantiles of the queue length. The load conditions are the same as in the previous figure, except for the fact that this time several values of b_1 are used for the WCT. Note that the curve slopes are dependent of the value of b_1.

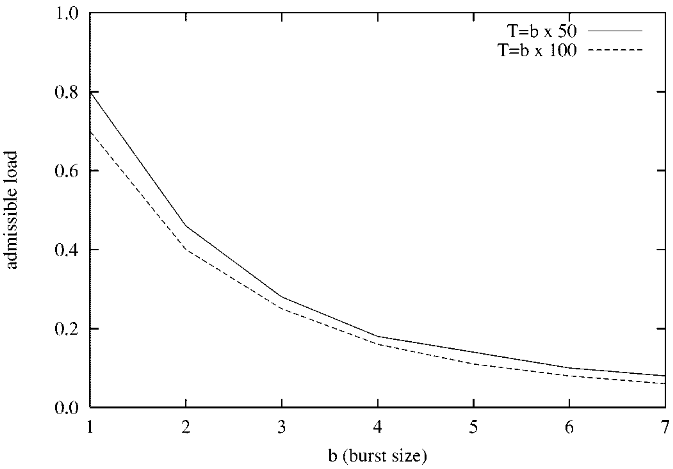

Fig. 6. Admissible load enabling a 10^{-6} quantile of the queue length CPDF of 14 cells to be obtained.

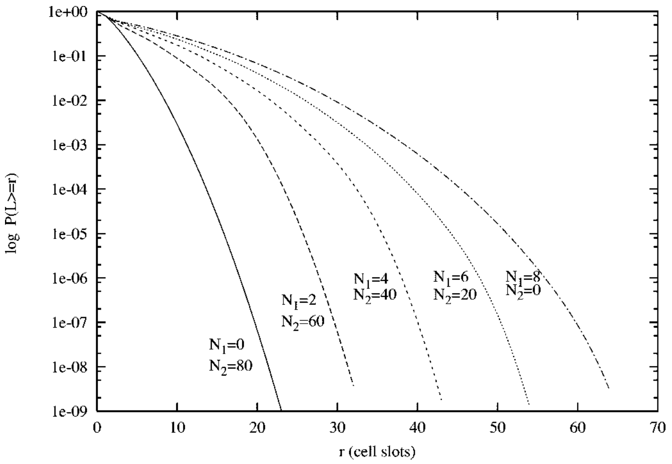

Fig. 7. CPDF for a single queue: heterogeneous traffic case, $\rho = 0.8$, . $T = 100$, $b_1 = 10$ and $b_2 = 1$.

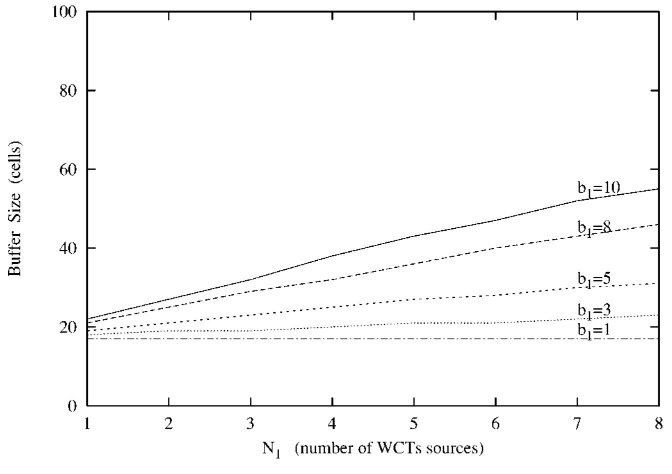

Fig. 8. 10^{-6} quantiles of the CPDF for different values of b_1. Same traffic conditions as in Fig. 7, except for the fact that several values of N_1 and b_1 are used for the WCT.

The models developed in this paper enable the effect of correlations between traffic sources to be studied in certain special cases. For example, we may assume a statistical multiplexer which receives traffic from N CBR sources of period T. If the

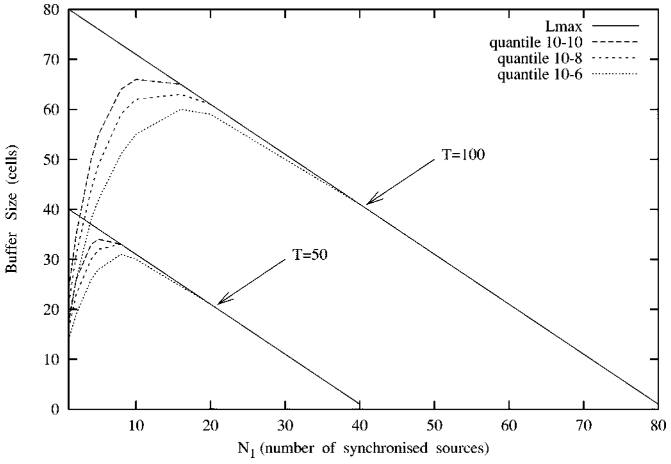

Fig. 9. Effect of synchronized sources on the queue length.

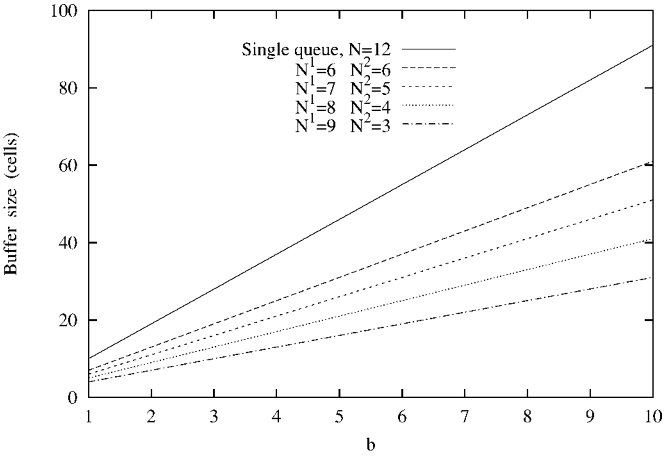

Fig. 10. 10^{-10} quantiles of the queue length CPDF in the root queue versus b. Two stages. $\rho = 0.8$ and $T = b \times 15$. 10^{-10} quantiles of the queue length CPDF for a single queue loaded with twelve sources is also shown.

sources are independent the buffer occupation can be obtained by using an $N \cdot D/D/1$ model. Nevertheless, if N_1 sources synchronize the emission of cells (form example, a terminal emits—whether intentionally or not—in a synchronized way in N_1 VCs) the multiplexer would receive the traffic from $N - N_1$ independent CBR sources, superimposed with the periodic emission of N_1 cells arriving in a burst.

In Fig. 9 we can observe the effect of this synchronization for $N = 40$, $T = 50$ and $N = 80$ and $T = 100$, and different values of N_1. The quantiles for 10^{-6}, 10^{-8} and 10^{-10} of the queue length CPDF are shown, as well as the maximum values of said queue length. The existence of these correlations can be seen to have a great impact on the quantiles of queue length. In this example a maximum for a value of N_1 roughly equal to $N/8$ is reached.

B. Two-Stage Multiqueue System

The second scenario to be studied is a two-stage multiqueue system loaded with a single class traffic sources [see Fig. 3(b)].

First, we will suppose that the second stage has two queues, which receive traffic from N^1 and N^2 WCT sources of burst length b cells and period $T = b \times 15$. The root queue does not receive direct traffic from external sources. Fig. 10 shows the 10^{-10} quantiles of the CPDF of the root queue length as a function of b for different values of N^1 and N^2. We maintain $N^1 + N^2 = 12$. For comparison purposes, the buffer size for a single queue loaded with twelve WCT sources is also shown.

It is seen that the greatest quantiles are obtained for the situation of balanced loads (e.g., $N^1 = N^2$). A possible explanation for this is the following: the larger queue length will appear when we have two simultaneous large busy periods in both second-stage queues. Balanced configurations make more likely this situation to happen. Fig. 11 shows the sum of the quantiles of queue length for both queues in the second stage. We can see an interesting fact: This sum remains almost constant for any pair of N^1 and N^2 values and is almost the same as the buffer size obtained for a single queue loaded with twelve sources (see Fig. 10). Note that the maximum queue length for a single queue loaded with N sources of burst length b is given by $L_{\max} = (N-1) \times b+1$. The sum of maximum queue length for queues 1 and 2 gives $L_{\max}^1 + L_{\max}^2 = (N^1 + N^2 - 2) \times b + 2 = (N-2) \times b + 2$, i.e., we can approximate the sum of maximum queue lengths by the maximum queue length of a single queue which multiplex the aggregate traffic.

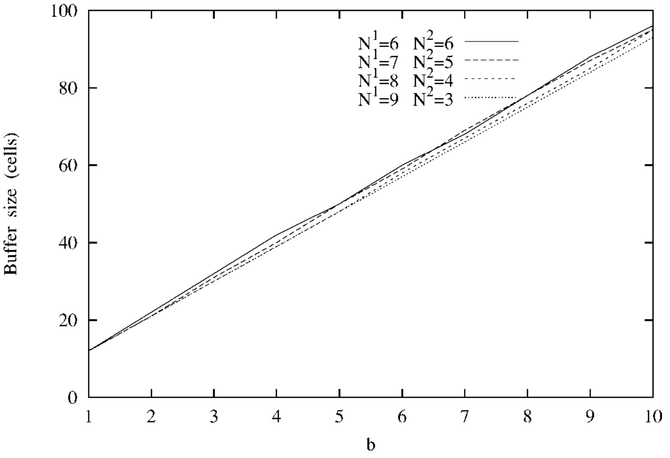

Fig. 11. Sum of the 10^{-10} quantiles of the queue length CPDF of the two queues of the second-stage versus b. Same traffic conditions as in Fig. 10.

Fig. 12 shows the queue length CPDF in the root queue when the second stage has $K = 2, 3, 4, 12$ queues. The total load is $\rho = 0.8$ and $b = 6$. The period of the sources is $T = 90$ and the load is balanced over the K queues (i.e., we have $N^k = (12/K)$ sources in each queue, $k = 1, \ldots, K$). As would be expected, the quantiles become greater as the K value rises. The worst results are obtained for $K = 12$, equivalent to the situation in which the 12 sources feed the root queue directly. However, it is observed that as soon as K exceeds 3 there are few differences in the results obtained.

C. Tandem Queueing System

The third and last scenario consists of a tandem queueing system with several stages [see Fig. 3(c)]. The queue for the third stage, numbered as queue 2, receives direct traffic from N^2 sources. The queue of the second stage, numbered as queue 1, receives traffic from N^1 sources, in addition to the output traffic from queue 2. The root queue receives direct traffic from

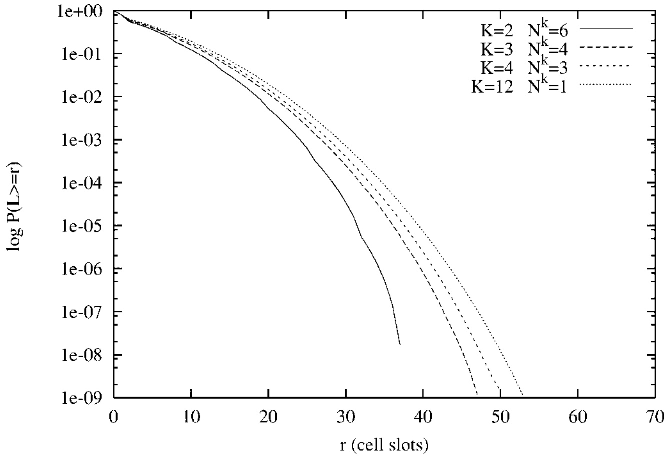

Fig. 12. Queue length distribution for 2, 3, 4, 12 queues in the second stage.

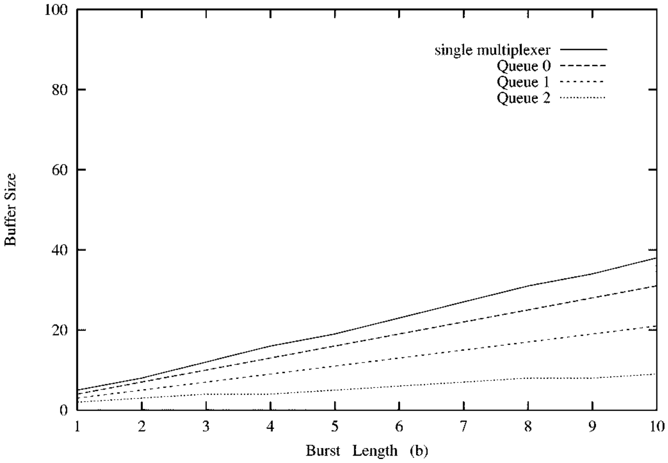

Fig. 13. Buffer size in queues 0, 1, and 2 as a function of b. Buffer size in the case of a single multiplexer loaded directly with the WCT sources is also shown.

N^0 sources in addition to the output traffic from queue 1. All the sources have the same period T.

First of all we shall study the case in which all the sources emit with the same burst length b. $N^2 = 2$, $N^1 = 3$ and $N^0 = 3$. Fig. 13 shows the quantile for the CPDF queue length in the three queues for different values of b. In all the cases we kept $T = b \times 10$. Fig. 13 also shows the 10^{-3} quantiles for the case of a single multiplexer loaded directly with 8 sources of similar characteristics (i.e., $N^1 = N^2 = 0$ and $N^0 = 8$). The effect of the WCT in the multi-stage system can be seen to be mitigated to a certain extent for the root queue.

VI. CONCLUSIONS

In this article the Beneš method has been applied to studying the multiplexing of periodic sources in statistical multiplexers connected in a tree network. Exact closed-form expressions were obtained.

There are two possible generalizations to the models studied. The first consists in eliminating the restriction that all the sources should share the same period. The second more interesting one consists in not imposing a tree topology for connecting multiplexers. In both cases it seems difficult to be able to obtain exact solutions. Nevertheless, our opinion is that the results obtained in this paper constitute a possible channel for allowing approximate results to be obtained for these more general situations.

The models developed were of use for studying several phenomena which may arise in packet networks which support real-time services, as is the case of ATM networks. The results clearly show the adverse effects that the presence of WCT may have on network performance, even when only a limited proportion of the traffic presents this anomalous behavior. In addition, they reveal the adverse effects that appear when several traffic sources are synchronized. In both cases, these effects are considerable not only when we have a single multiplexing stage but also (although mitigated to some extent) in multistage configurations.

Several mechanisms (e.g., shapers) have been proposed that are able to solve these potential problems. Logically, these mechanisms involve an additional cost. We believe that the results given here may be useful for assessing the suitability or otherwise of introducing these mechanisms into ATM traffic control.

APPENDIX
BENES METHOD APPLIED TO THE TIME-SLOTTED G/D/1 QUEUE

Let us consider a discrete-time multiplexer with a service time equal to one time-slot. Slot 0 is taken as being an arbitrary time slot. The Beneš formula is based on the observation that event $\{L_0 > x\}$ can be partitioned into disjoint events $\{\mathcal{A}(t) = t + x,\, L_{-t} = 0\}$ for $t \geq 1$ (see [13] for a proof of this):[7]

$$\begin{cases} \bigcup_{t=1}^{\infty} \{\mathcal{A}(t) = t + x,\, L_{-t} = 0\} = \{L_0 > x\} \\ \{\mathcal{A}(t) = t + x,\, L_{-t} = 0\} \cap \\ \quad \{\mathcal{A}(t') = t' + x,\, L_{-t'} = 0\} = \emptyset \text{ for } t \neq t' \end{cases} \tag{23}$$

By applying the total probability formula the following expression for $P\{L_0 > x\}$ can be obtained:

$$P\{L_0 > x\} = \sum_{t=1}^{\infty} P\{\mathcal{A}(t) = t + x,\, L_{-t} = 0\}. \tag{24}$$

If the system is periodic and stable [12], [13], the queue is empty at least in some slot in every period and switching off the cell arrivals for $t \leq -T$ does not modify the queue length at time $t = 0$ [Fig. 14(a) and (b)]. For this modified system $P\{\mathcal{A}(t + x) = t + x,\, L_{-t} = 0\} = 0$ for $t > T$ and hence the sum in (24) can be truncated, which leads to

$$P\{L_0 > x\} = \sum_{t=1}^{T} P\{\mathcal{A}(t) = t + x,\, L_{-t} = 0\}. \tag{25}$$

In the particular model considered in this paper, in which we have burst arrivals, we can also switch off the cells arriving in $(-T, 0]$ but that are originated by bursts that start at $t \leq -T$ [see Fig. 14(c)]. The reason is that the first empty slot in the period necessarily comes only after these bursts have ended, and consequently they do not have influence on the queue length at time $t = 0$. *Throughout this paper we consider this arrival*

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[7] $\mathcal{A}(t)$ and L_t are defined in Section II.

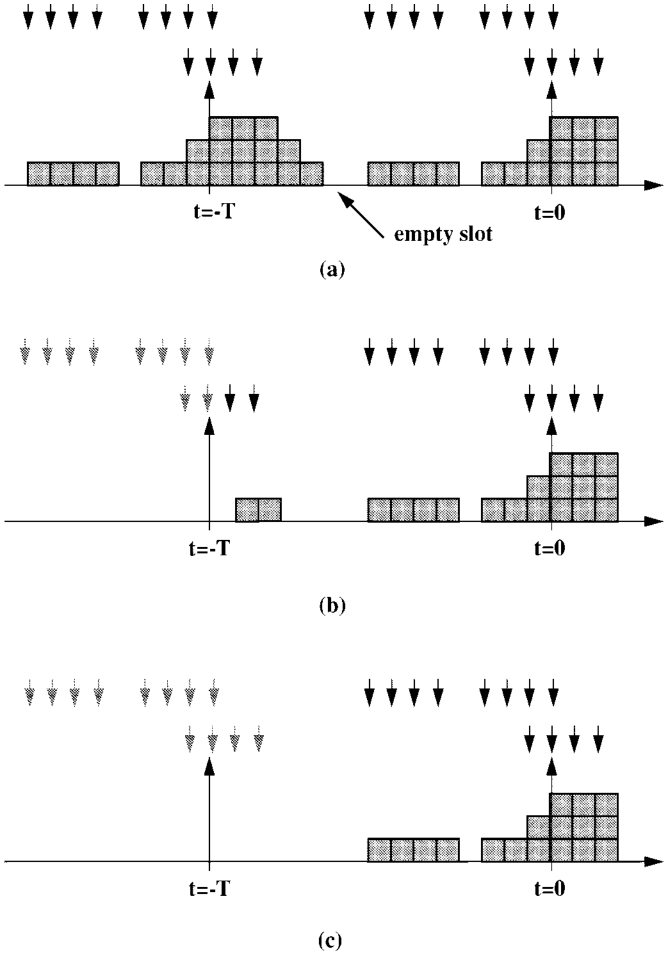

Fig. 14. In periodic and stable systems the arrival pattern can be modified while L_0 remains unchanged. (a) Original periodic system. (b) The arrival pattern consists solely of the arrivals of the original system in $(-T, 0]$, and not the full periodic system. (c) In the case of burst arrivals the wrapped tails of the bursts originated for $t \leq -T$ can also be neglected.

pattern, in which the cells of bursts that start for $t \leq -T$ *are neglected.*

As explained in [12, pcd $. 399], (25) can have two different interpretations. On the one hand, it is the stationary complementary distribution function (CDF) of the queue length for periodic and stable systems. On the other hand, it gives the CDF of queue length for transient systems starting from an empty state at time $-T$, even in the case of unstable systems.

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